Matric Revision!

Paper **1** 150 marks

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Maths

Maths 1 2 3

Gr 12

CAPS Maths matric revision book for learners, parents and teachers

1 2 3 Offers a clear and systematic plan to achieve success with the obtaining of 300 marks for Maths Paper 1 and Paper 2 in Gr 12

Achieve success as easy as 1 2 3

1 **Theory and How-to**

2 **Revision and consolidation**

3 **Exam questions**

Description of Maths 1 2 3 Gr 12 Matric Revision Books

1 2 3 is for preparing for the National Maths Examination for Grade 12.

The examination consists of 2 compulsory papers: **Paper 1 and Paper 2.** Three hours are allocated to each paper and each consists of **150 marks**.

1 2 3 are two CAPS Matric Revision books for learners, parents and teachers.

1 2 3 **takes** you from the **basic theory** to **typical Grade 12 final exam questions** in the minimum time.

The **headings** with possible **mark allocations** ☐ are your guide throughout the book.

Contents of Maths 1 2 3 Gr 12 Paper 1 Matric Revision Book

Module 3 Functions and Inverse Functions

pacesetter!

35±3 pages ➡ ⬅ 35±3 marks

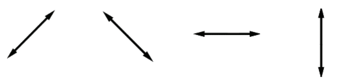
- **Do everything. Mark each question that you struggle with so that you can return and attempt it later.**
- ☐ **Indicates the possible mark allocation**
- 3 **may represent 1% of matric**

Theory | Concepts and facts in functions and inverse functions

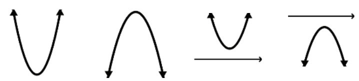
Revision of previous grades 16±4

Recognise the equation and form of the function and key properties

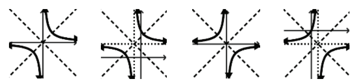
- $\boldsymbol{y = mx + c}$ **for straight lines**, both $\boldsymbol{x}$ **and** $\boldsymbol{y}$ are **to the power of 1,** a **linear equation**.



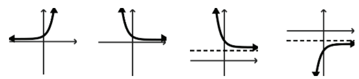
- $\boldsymbol{y = a(x+p)^2 + q}$ **for** the **parabolic-** or **quadratic equation,** $\boldsymbol{x}$ is **squared**.



- $\boldsymbol{y = \dfrac{a}{x+p} + q}$ **for** the **hyperbola,** $\boldsymbol{x}$ is **in the denominator** of the fraction.



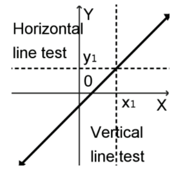
- $\boldsymbol{y = ab^{x+p} + q}$ **for exponential equations,** $\boldsymbol{x}$ is **an exponent**.



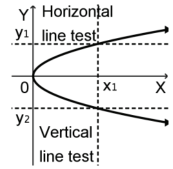
Functions and non-functions

Determine the type of relation — **If** a **vertical line** parallel to the y-axis **intersects the graph more than once** the **relation is not** a **function**.

A one-to-one mapping, a function of x

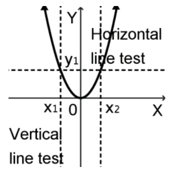


A one-to-one mapping, not a function



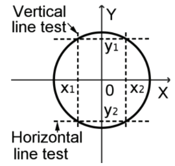
A many-to-one mapping, a function of x

∴ **one many-to-one mapping** will be a **function**



A many-to-many mapping, not a function

∴ **one many-to-many mapping** will **not** be a **function**



Translation and reflection of a function

Any transformation involves **changing both the equation and the graph of a function**.
A change in the equation will result in a change in the graph and vice versa.

	Function notation	Effect on equation	Effect on graph
Translation or shift Change in position	$f(x)+q$	A constant q is added to the function formula. $(x;y)\to(x;y+q)$	**Vertical shift** Graph shifts up when $q>0$ and down when $q<0$.
	$f(x+p)$	x is replaced with $x+p$. $(x;y)\to(x+p;\ y)$	**Horizontal shift** Graph shifts left when $p>0$ and right when $p<0$.
Reflection Change in orientation	$-f(x)$	The sign of every term in the formula changes.	**Reflection in the $\boldsymbol{y=0}$ line** Graph reflects about the ***x*-axis**.
	$f(-x)$	x is replaced with $-x$.	**Reflection in the $\boldsymbol{x=0}$ line** Graph reflects about the ***y*-axis**.
	$f(y)$	x is replaced with y and y is replaced with x. $(x;y)\to(y;x)$	**Reflection in the $\boldsymbol{y=x}$ line**
Vertical stretch or compression Change in shape	$af(x)$	Multiply the function with a constant a. If a is negative, the graph is reflected in the x-axis.	Graph is **stretched away from** the ***x*-axis** if $\boldsymbol{a>1}$ **or** $\boldsymbol{a<-1}$ and **squashed towards** the ***x*-axis** if $\boldsymbol{-1<a<1}$.

Revision and consolidation

Interpretation of graphs

10±2

Inequalities and property of roots

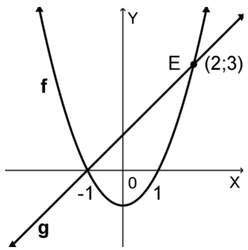
- **Graphically**

a Use the graphs of *f* and *g* to determine the value of *x* if

1 $f(x)=g(x)$ **2** $f(x)>g(x)$ **3** $g(x)>f(x)$

4 $f(x)\cdot g(x)=0$ **5** $f(x)\cdot g(x)<0$ **6** $f(x)\cdot g(x)>0$

7 $\frac{f(x)}{g(x)}<0$ **8** $x\cdot f(x)<0$ **9** $x\cdot f'(x)>0$



Solution

1 $f(x)=g(x)$
$\therefore \boldsymbol{x}=-1$ **or** $\boldsymbol{x}=2$
- $\boldsymbol{f(x)}$ **and** $\boldsymbol{g(x)}$ **intersect**

2 $f(x)>g(x)$
$\therefore \boldsymbol{x}<-1$ **or** $\boldsymbol{x}>2$
- $\boldsymbol{f(x)}$ **lies above** $\boldsymbol{g(x)}$**, answer is excluded**

3 $g(x)>f(x)$
$\therefore -1<\boldsymbol{x}<2$
- $\boldsymbol{g(x)}$ **lies above** $\boldsymbol{f(x)}$**, answer is excluded**

4 $f(x)\cdot g(x)=0$
$\therefore \boldsymbol{x}=-1$ **or** $\boldsymbol{x}=1$
- $\boldsymbol{f(x)=0}$ **or** $\boldsymbol{g(x)=0}$**, *x*-intercepts of** $\boldsymbol{f(x)}$ **and** $\boldsymbol{g(x)}$

5 $f(x)\cdot g(x)<0$
$\therefore \boldsymbol{x}<1$ **or** $\boldsymbol{x}\neq -1$
- **Signs differ,** $+.-=-$**, one graph above the *x*-axis and the other one under the *x*-axis, answer is excluded**

6 $f(x)\cdot g(x)\geq 0$
$\therefore \boldsymbol{x}\geq 1$
- **Signs are the same,** $+.+=+$ **or** $-.-=+$**, both graphs above the *x*-axis or both under the *x*-axis, answer is included**

7 $\frac{f(x)}{g(x)}<0$
$\therefore \boldsymbol{x}<1$ **or** $\boldsymbol{x}\neq -1$
- **Signs differ,** $+.-=-$**, one graph above the *x*-axis and the other one under the *x*-axis, answer is excluded**

The linear function and its inverse

8±2

Theory | The linear function $y=ax+q$ or $y=mx+c$, the straight line

From now on we only use ***m* and *c***, $\boldsymbol{m}=\boldsymbol{a}$ **and** $\boldsymbol{c}=\boldsymbol{q}$.

Equations

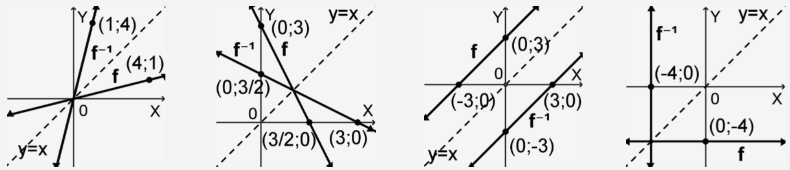
$\boldsymbol{m}>\boldsymbol{0}$ ↗ **or** $\boldsymbol{m}<\boldsymbol{0}$ ↘

Standard equation	$y=mx+c$ (Shifts up and down)		• ***m*** **influences a reflection in the *x*-axis and steepness** ***c*** **is the *y*-intercept**
For lines through the origin	$y=mx;\ m\neq 0$		• **Lines passes through** $(0;0)$**,** $\boldsymbol{c}=\boldsymbol{0}$
For lines | *x*-axis	$y=$ number	↔	• **Cuts the *y*-axis at that number but never cuts the *x*-axis,** $\boldsymbol{m}=\boldsymbol{0}$
For lines | *y*-axis	$x=$ number	↕	• **Cuts the *x*-axis at that number but never cuts the *y*-axis, *m* undefined**

Inverse of the straight line

8

- **The inverse of a straight line is a function**.
  - **Replace** x with y and y with x.
  - f and f^{-1} are **symmetrical** iro the line $\boldsymbol{y = x}$.
  - The **domain of *f*** becomes the **range of f^{-1}**.
  - The **range of *f*** becomes the **domain of f^{-1}**.



a Given $h(x) = 2x - 3$ for $-2 \le x \le 4$. The x-intercept of h is Q.

1 Determine the co-ordinates of Q.

2 Write down the domain of h^{-1}.

3 Sketch the graph of h^{-1}, clearly indicating the y-intercept and the end points.

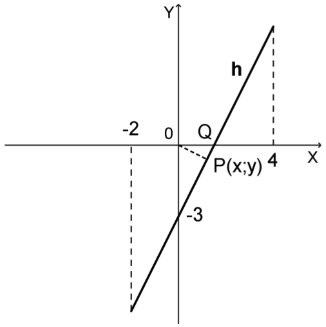
4 For which value(s) of x will $h(x) = h^{-1}(x)$?

5 $P(x;y)$ is the point on the graph of h which is closest to the origin. Calculate the distance OP.

6 Given $h(x) = f'(x)$ where f is a function defined for $-2 \le x \le 4$.

6.1 Explain why f has a local minimum.

6.2 Write down the value of the maximum gradient of the tangent to the graph of f.



b Given $y = 1$.

1 Sketch the graph of the inverse of $y = 1$.

2 Is the inverse of $y = 1$ 'n function? Motivate your answer.

Solution

a 1 For *x*-intercept

$0 = 2x - 3$

$x = \frac{3}{2}$

$\mathbf{Q\left(1\frac{1}{2};0\right)}$

2 $h(x) = 2x - 3$

$h(-2) = 2(-2) - 3$

$= -7$

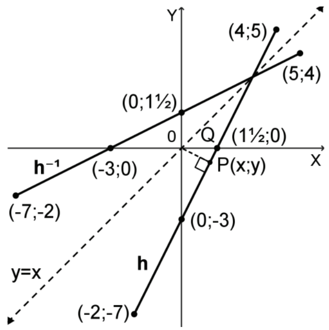
$h(4) = 2(4) - 3$

$= 5$

$W_h: \ -7 \le y \le 5$

$\therefore \mathbf{D_h^{-1}: \ -7 \le x \le 5}$

3



4 h, h^{-1} and $y = x$ intersect in one point

$\boldsymbol{y = x}$ ... (1)

$y = 2x - 3$ ... (2)

$\therefore \boldsymbol{x} = 2x - 3$ ... (1) = (2)

$\therefore \boldsymbol{x = 3}$

5 P will be the closest to the origin if OP is perpendicular to h

For *h* $y = 2x - 3$ ... (1)

$m = 2$

$m_{\perp} = -\frac{1}{2}$

$\therefore M_{OP} = -\frac{1}{2}$ and $c = 0$

Eqn of OP

$y = -\frac{1}{2}x$ ... (2)

$\therefore 2x - 3 = -\frac{1}{2}x$ ... (1) = (2)

$2\frac{1}{2}x = 3$

$\left(\times\frac{\mathbf{2}}{\mathbf{5}}\right) \ x = \frac{6}{5}$ ... in (2)

$\therefore y = -\frac{1}{2}\left(\frac{6}{5}\right)$

$= \frac{-3}{5}$

$\therefore P\left(\frac{6}{5};\frac{-3}{5}\right)$

$OP^2 = (x_P)^2 + (y_P)^2$

$= \left(\frac{6}{5}\right)^2 + \left(\frac{-3}{5}\right)^2$

$= \frac{36}{25} + \frac{9}{25}$

$= \frac{45}{25}$

$\therefore \mathbf{OP} = \sqrt{\frac{45}{25}}$

$= 1{,}3416$

... $\approx$ **1,34 units**

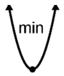
The parabolic function and its inverse, the root function

8±2

Theory | The parabolic function $y = a(x+p)^2 + q$, the parabola

Equations

$a > 0$



or **$a < 0$**

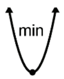
$\left(-\frac{b}{2a}; f\left(\frac{-b}{2a}\right)\right)$ $\left(-\frac{b}{2a}; f\left(\frac{-b}{2a}\right)\right)$

Standard form $y = ax^2 + bx + c$

- **a, b and c are constants and $a \neq 0$**

  c is the y-intercept

$a > 0$



or **$a < 0$**

$(-p; q)$ $(-p; q)$

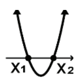
Turning point formula $y = a(x+p)^2 + q$

Shifts right and left ⇄ Shifts up and down ⇅

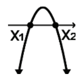
- **a influences form, reflection in the x-axis and steepness**

  q is the y-intercept

$a > 0$



or **$a < 0$**



x-intercept formula $y = a(x - x_1)(x - x_2)$

- **$x = x_1$ or $x = x_2$ are the x-intercepts**

Inverse of the parabola

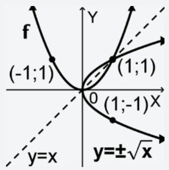
8

- **The inverse of the parabola is not a function**.
  - $f(x) = x^2$; $x \in \mathbb{R}$ and $y \geq 0$

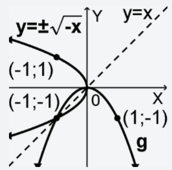
    Inverse $y = \pm\sqrt{x}$; $x \geq 0$ and $y \in \mathbb{R}$
  - $g(x) = -x^2$; $x \in \mathbb{R}$ and $y \leq 0$

    Inverse $y = \pm\sqrt{-x}$; $x \leq 0$ and $y \in \mathbb{R}$
  - The **graph of the inverse of f** is thus in **quadrant 1 and 4** and **the inverse of g** is in **quadrant 2 and 3**.

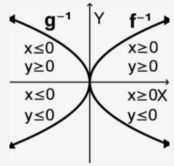
The graph of the inverse of f is $y = \pm\sqrt{x}$ and is a parabola that lies on its side, to the **right of the y-axis** and symmetrical about the x-axis.
The inverse of a parabola is **not a function**.



The graph of the inverse of g is $y = \pm\sqrt{-x}$ and is a parabola that lies on its side, to the **left of the y-axis** and symmetrical about the x-axis.
The inverse of a parabola is **not a function**.

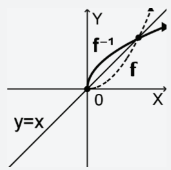


- **If** you see a $\sqrt{\ }$, it is the **inverse of a parabola** and graphically it is going to be one of the **four legs of a parabola** lying on its side.

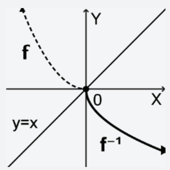


- **The restricted parabola and its inverse is a function**.

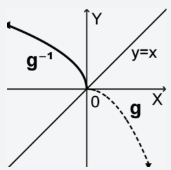
f : $y = +\sqrt{x}$



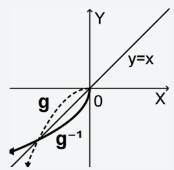
f : $y = +\sqrt{-x}$



g : $y = -\sqrt{x}$



g : $y = -\sqrt{-x}$



a Given $g(x)=\frac{-x^2}{4}$.

1 Determine the equation of the inverse of g in the form $y=...$

2 On the same set of axes, draw the graphs of $y=g(x)$ and its inverse.

3 Write down the domain and range of g.

4 Hence, write down the domain of the inverse of g.

b Given $h(x)=\frac{4}{9}x^2$.

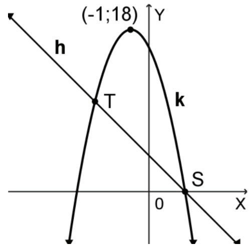
1 Write down the equation of g if g is the reflection of h about the y-axis.

2 Write down the equation of k if h is translated two units down to obtain k.

3 Determine the equation of $h^{-1}(x)$. Leave your answer in in the form $y=...$

4 The inverse of h is not a function. Restrict the domain of h such that h^{-1} is a function. Sketch the restricted graph of h and $h^{-1}(x)$ on the same system of axes.

c Sketched are the graphs of $k(x)=ax^2+bx+c$ and $h(x)=-2x+4$. Graph k has a turning point at $(-1;18)$. S is the x-intercept of h and k. Graphs h and k also intersect at T.



1 Calculate the co-ordinates of S.

2 Determine the equation of k in the form $y=a(x+p)^2+q$.

3 Determine the co-ordinates of the turning point of p if $p(x)=k(3x)$.

4 If $k(x)=-2x^2-4x+16$, determine the co-ordinates of T.

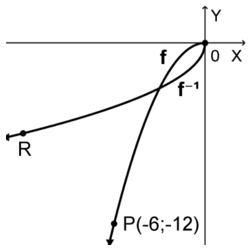
5 Determine the value(s) of x for which $k(x)<h(x)$.

6 It is further given that k is the graph of $g'(x)$.

6.1 For which values of x will the graph of g be concave up?

6.2 Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection.

d In the diagram, the graph of $f(x)=ax^2$ is drawn for $x\le 0$. The graph of f^{-1} is also drawn. $P(-6;-12)$ is a point on f and R is a point on f^{-1}.



1 Is f^{-1} a function? Motivate your answer.

2 If R is the reflection of P in the line $y=x$, write down the co-ordinates of R.

3 Calculate the value of a.

4 Write down the equation of f^{-1} in the form $y=...$

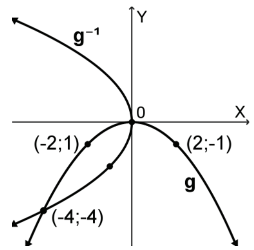
Solution

a 1 $x=\frac{-y^2}{4}$

$\therefore y^2=-4x$

$\therefore \boldsymbol{y=\pm\sqrt{-4x}}$ and $\boldsymbol{x\le 0}$

2



3 $\boldsymbol{x\in\mathbb{R};\ y\in(-\infty;0]}$ **or** $\boldsymbol{y\le 0}$

4 $\boldsymbol{x\in(-\infty;0]}$ **or** $\boldsymbol{x\le 0}$

b 1 $g(x)=-h(x)$ • $\boldsymbol{x\to -y}$

$=\boldsymbol{\frac{-4}{9}x^2}$

2 $k(x)=h(x)-2$

$=\boldsymbol{\frac{4}{9}x^2-2}$

3 For h $y=\frac{4}{9}x^2$

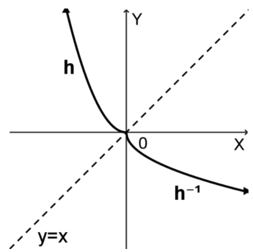
For h^{-1} $x=\frac{4}{9}y^2$

$\therefore y^2=\frac{9}{4}x$

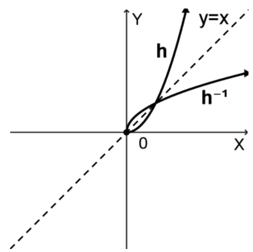
$\therefore \boldsymbol{y}=\pm\sqrt{\frac{9}{4}x};\ x\ge 0$

$=\pm\boldsymbol{\frac{3}{2}\sqrt{x};\ x\ge 0}$

4 $h(x)$ with $x\le 0$



$h(x)$ with $x\ge 0$



Grade 12 The fundamental counting principle, permutations and probability problems

7±3

Symbols and terminology of the fundamental counting principle and factorials

The fundamental counting principle The fundamental counting principle states that if one event has $\boldsymbol{a}$ possible outcomes, a second event has $\boldsymbol{b}$ possible outcomes and a third event has $\boldsymbol{c}$ possible outcomes, then there are $\boldsymbol{a \times b \times c}$ possible outcomes for the three events together.

How many different outfits are possible with the following combination of clothes?

Trousers	**Shirts**	**Shoes**
Grey	Blue	Trainers
Black	Purple	Flip-flops
	Red	

Solution

Using the fundamental counting rule

$\underset{\text{trousers}}{2} \times \underset{\text{shirts}}{3} \times \underset{\text{shoes}}{2} = 12$ options

With repetition $\boldsymbol{n^r}$ **For independent recurring events** ie when a ball gets replaced in a bag the **number of possibilities** where n terms can fill r positions $= \boldsymbol{n \times n \times n \ldots \times n\,(r \text{ times}) = n^r}$ **possibilities**.

Without repetition $\boldsymbol{n!}$ For dependent events the first choice influences the second choice, etc, eg when a ball is not returned to a bag.

The **number of possible ways** n different terms can be arranged, **without repetition**, will be $\boldsymbol{n! = n(n-1)\times(n-2)\times\ldots\times 1}$. This is read $\boldsymbol{n}$ **factorial** and is written as $\boldsymbol{n!}$

$4! = 4\times3\times2\times1$

$0! = 1$

Probability The **fundamental counting principle** and factorials are used to find $n(S)$, the **number of options** in the sample space, where $\mathbf{P(A)} = \dfrac{\boldsymbol{n}\mathbf{(A)}}{\boldsymbol{n}\mathbf{(S)}}$.

Revision and consolidation on Grade 11

Probability problems

8±3

Probability rules

4

- $\mathbf{0 \le P(A) \le 1}$
- $\mathbf{P(A)} = \dfrac{\boldsymbol{n}\mathbf{(A)}}{\boldsymbol{n}\mathbf{(S)}}$
  - **For any two events A and B** $\Rightarrow \mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$
  - **Mutually exclusive** $\Rightarrow \mathbf{P(A \text{ and } B) = 0}$
    $\therefore \mathbf{P(A \text{ or } B) = P(A) + P(B)}$
  - **Independent** $\Rightarrow \mathbf{P(A \text{ and } B) = P(A) \times P(B)}$
    - **And** means **multiply** $\times$
    - **Or** means **add** $+$
- **Complementary events** $\Rightarrow \mathbf{P(A \text{ or } A') = P(A) + P(A') = 1}$
  $\mathbf{P(A') = 1 - P(A)}$

a A, B, C and D are 4 events in a sample space. $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{2}{5}$, $P(D) = k$ and $P(C \text{ or } D) = \frac{7}{10}$.

1 Determine $P(A \text{ or } B)$ if A and B are mutually exclusive events.
2 Determine $P(A \text{ or } B)$ if A and B are independent events.
3 For which value of k will C and D be mutually exclusive?
4 For which value of k will C and D be independent?

b If $P(A') = 0{,}45$ and $P(B) = 0{,}35$, calculate $P(A \text{ or } B)$ if

1 A and B are mutually exclusive events
2 A and B are independent events.

Venn-diagrams

4

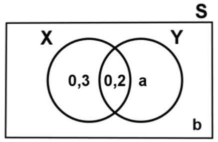
- **Apply probability rules**.

a **1** N and M are two events. $P(N)=0{,}3$, $P(M)=0{,}4$ and $P(M \text{ or } N)=0{,}6$.

1.1 Sketch a Venn-diagram to represent the events

1.2 Are the events N and M independent?
Motivate your answer by showing all relevant calculations.

2 The Venn-diagram refers to probabilities. If X and Y are independent events, find the values of *a* and *b*. Show all your calculations.



3 Events A and B are independent. $P(A)=0{,}4$ and $P(B)=0{,}25$.

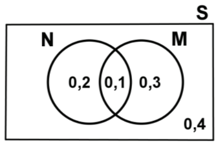
3.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region.

3.2 Determine $P(A \text{ or not } B)$.

Solution

1.1

$$\mathbf{P(M \text{ or } N)=P(N)+P(M)-P(M \text{ and } N)}$$
$$0{,}6=0{,}3+0{,}4-P(M \text{ and } N)$$
$$\therefore P(M \text{ and } N)=0{,}3+0{,}4-0{,}6$$
$$=\mathbf{0{,}1}$$



1.2

$$P(N)\times P(M)=0{,}3\times 0{,}4$$
$$=0{,}12$$
$$P(M \text{ and } N)=0{,}1$$
$$\therefore \mathbf{P(M \text{ and } N)\neq P(M)\times P(N)}$$

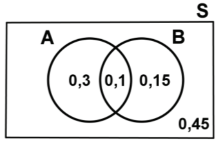
$\therefore$ **No, M and N are not independent**

2

$$P(X)\cdot P(Y)=P(X\cap Y)$$
$$\therefore (0{,}5)(a+0{,}2)=0{,}2$$
$$\therefore 0{,}5a+0{,}1=0{,}2$$
$$\therefore 0{,}5a=0{,}1$$
$$\therefore \boldsymbol{a}=\mathbf{0{,}2}$$
$$\therefore \boldsymbol{b}=1-0{,}7$$
$$=\mathbf{0{,}3}$$

3.1

$$\mathbf{P(A \text{ and } B)=P(A)\times P(B)}$$
$$=0{,}4\times 0{,}25$$
$$=\mathbf{0{,}1}$$



3.2

$$P(\text{not } B)=0{,}3+0{,}45$$
$$=0{,}75$$
$$\mathbf{P(A \text{ or not } B)=P(A)+P(\text{not } B)-P(A \text{ and not } B)}$$
$$=0{,}4+0{,}75-0{,}3$$
$$=\mathbf{0{,}85}$$

2-way contingency diagrams

4

- **Note $(A\cap B)$ for any 2 events from the diagram**.
- $\mathbf{P(A)=\dfrac{\boldsymbol{n}(A)}{\boldsymbol{n}(S)}}$
- $\mathbf{P(A\cap B)=\dfrac{\boldsymbol{n}(A\cap B)}{\boldsymbol{n}(S)}}$
- **Use probability rules**.

a A survey on their holiday preferences was done on 180 staff members. The options they could choose from were going to the coast, visit a game park or stay at home. The results are presented in the table below.

	Coast	Game park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

1 Determine the probability that a randomly selected staff member
1.1 is male **1.2** prefers not visiting a game park.

2 Are the events being a **male** and **staying at home** independent events? Motivate your answer with relevant calculations.

4 $n(S)=9$

No 1 **or** the same $=\{(1,1)(2,2)(2,3)(3,2)(3,3)\}$ • **Union**

$\therefore n(\text{no 1 or the same})=5$

$\therefore \mathbf{P(\text{no 1 or the same})=\frac{5}{9}}$ **or 0,56 or 56%**

Tree-diagrams

4

A tree diagram is used to **illustrate compound events**.

- Events that **follow each other** can be **dependent or independent**.
  - **Dependent** events, where the outcome of the second event is affected by the outcome of the first, will be illustrated by without replacement examples.
  - **Independent** events, where the outcome of the second event is not affected by the outcome of the first, will be illustrated by with replacement examples.
- **Marks are allocated for**
  - **drawing** the correct **number of branches**
  - **writing down** events or **outcomes** A, B, C, etc and the **probability on the branches**
  - **writing down** the **compounded outcomes** for **each branch** ADD, ADE, AED, etc.

a A packet of sweets has 3 pink, 2 green and 5 blue sweets. Two sweets are removed from the packet, one at a time, without replacement.
1 Draw a tree diagram to show all possible outcomes. Indicate on your diagram the probability associated with each branch of the tree diagram.
2 Determine the probability that
2.1 both sweets are blue **2.2** a green and a pink sweet are selected.

b There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to the bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%. Calculate how many orange balls are in the bag.

c There are **6 red** cards and **1 black** card in a box. Busi and Khanya take turns to draw a card at random from the box, with Busi being the first one to draw. The first person who draws the black card will win the game. Assume that the game can go on indefinitely. If the cards are drawn **with** replacement, determine the probability that Khanya will win, showing all calculations.

Probability problems

7±3

Fundamental counting principles

3

With repetition n^r	**For independent recurring events** ie when a ball gets replaced in a bag the **number of possibilities** where n terms can fill r positions $= \mathbf{n\times n\times n\ldots\times n\ (r\ \text{times})=n^r}$ **possibilities**.
Without repetition $n!$	For dependent events the first choice influences the second choice, etc, eg when a ball is not returned to a bag. The **number of possible ways** n different terms can be arranged, **without repetition**, will be $\mathbf{n!=(n-1)\times(n-2)\times\ldots\times 1}$. This is read n **factorial** and is written as $\mathbf{n!}$ $4!=4\times3\times2\times1$ $0!=1$
Probability	The **fundamental counting principle** and factorials are used to find $n(S)$, the **number of options** in the sample space, where $\mathbf{P(A)=\frac{n(A)}{n(S)}}$.

a Nametso may choose DVDs from three categories as listed in the table below.

Drama	**Romance**	**Comedy**
Last Hero	One Heart	Laughing Dragon
Midnight	You and Me	Falling Down
Stranger Calls	Love Song	Sitting on the Stairs
Missing in Action	Bird's First Nest	
Only 40 Seconds Left		

1 Nametso must choose 1 DVD from the Drama category. What is the probability that she will choose **Midnight**?
2 How many different selections are possible if her selection must include 1 drama, 1 romance and 1 comedy?
3 Calculate the probability that she will have **Last Hero** and **Laughing Dragon** as part of her selection in **2**.

Arrangement in a row

4

With repetition n^r	The **number of ways** where ***n* terms can fill *r* positions, with repetition**, is given by $= n \times n \times n \ldots \times n\,(r \text{ times}) = n^r$ possibilities.
Without repetition/ replacement $n!$	The **number of arrangements possible** of one group of distinguishable objects is given by $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$ • $0! = 1$
Identical items	The **number of different ways** that n items **with repetition** can be arranged, if some of them are identical, is given by $\frac{n!}{s_1!\,s_2!\ldots}$ where s_1 and $s_2 \ldots$ represent the individual identical items.

Non-identical items

3

a **1** Given the digits 3; 4; 5; 6; 7; 8 and 9.

1.1 Calculate how many unique 5-digit codes can be formed using the digits above, if
1.1.1 the digits may be repeated **1.1.2** the digits may not be repeated.

1.2 How many unique 3-digit codes can be formed using the above digits, if digits may be repeated, the code is greater than 400 but less than 600 and the code is divisible by 5.

2 The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5000?

b The letters of the word **DECIMAL** are randomly arranged into a new word, also consisting of seven letters. How many different arrangements are possible if

1 letters **may be repeated**
2 letters **may not be repeated**
3 the arrangement must start with a vowel and end in a consonant and **no repetition of letters is allowed**?

c The letters in the word **JOHAN** are arranged in any order without repetition. What is the probability that the word Johan will start with the letter J and end with the letter A?

2.2 Number of ways $= 1 \times 5! \times 1$
$= \mathbf{120}$

• | K | | | | | | P |
|---|---|---|---|---|---|---|

2.3 4 coastal cities
$\therefore$ Number of ways $= 4! \times (3+1)!$
$= \mathbf{576}$

• $\boldsymbol{m!(d+1)!}$

Examination questions | Probability

Exam questions 1

a **1** $P(A) = \frac{1}{4}$ and $P(A \text{ or } B) = \frac{1}{3}$. Find $P(B)$ as a simplified fraction for each of the following if

1.1 A and B are mutually exclusive events (3)
1.2 A and B are not exclusive events, but A and B are independent events. (5)

2 Consider the word **EINSTEIN**.

2.1 How many letter arrangements are possible if you use all the letters? (2)
2.2 What is the probability that identical letters will always be grouped together? (4)

b At registration time at the University of South Africa, a student can choose from the following three subject groups for the B.Com course he wants to enrol in.

Subject group A	Subject group B	Subject group C
Accounting	Financial Management	Applied Mathematics
Auditing	Financial Mathematics	Physics
Taxation	Office Management	Chemistry
Statistics	Psychology	Biochemistry
Mathematics		Physiology

1 If a student decides to choose one subject from Group A and one subject from Group B, how many different combinations are possible? (1)

2 If the student chooses one subject from each of the three subject groups but knows that he will not choose Biochemistry and Chemistry, how many combinations will there be to choose from? (2)